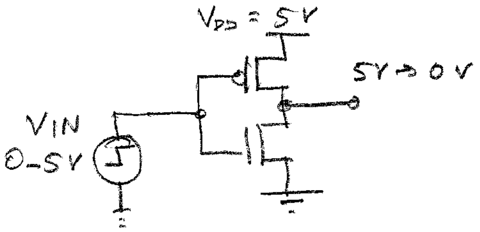
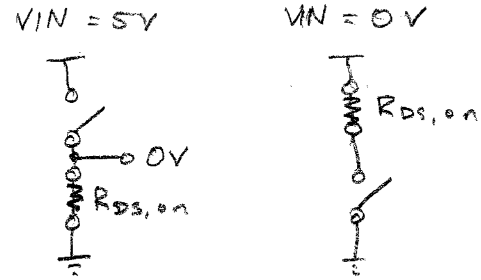
Mitchell Kline

EE105 Lecture: MOSFETs (Part 2), 10/11/2012

Review from last time

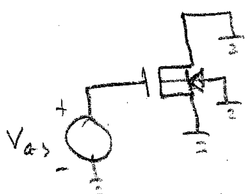
1. MOSFET as switch



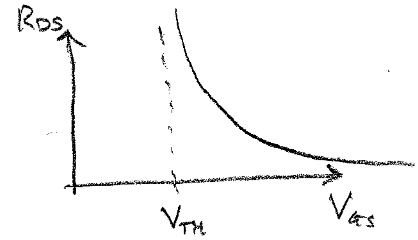


$$R_{DS,on} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{TH})}$$

2. MOSFET as controlled resistor

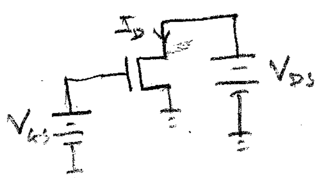


$$R_{DS} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} \Rightarrow$$



$$I_D = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) V_{DS}$$

3. MOSFET as current source

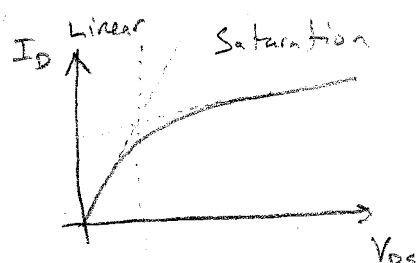


Pinch off requirement: $V_{GD} < V_{TH} \Rightarrow V_{DS} > V_{GS} - V_{TH}$

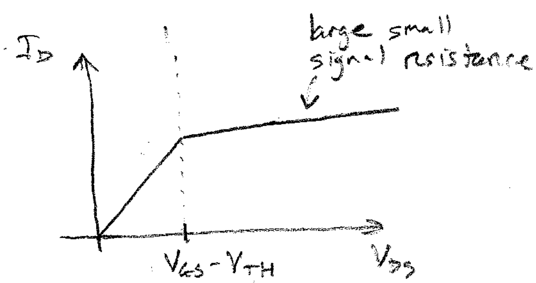
Saturation current: $I_{DSAT} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$

channel length modulation

② + ③ form piecewise linear model of I_D vs. V_{DS}



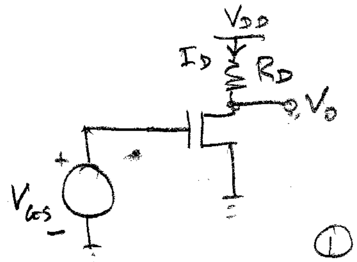
Actual



Model

4. MOSFET as amplifier.
   Key idea: control drain current with input voltage

Today:
- look at two topologies, common source + common drain
- design of an amplifier for a condenser microphone



- Assume saturation, otherwise I_D is also a <u>strong</u> function of V_o.

① $V_o = V_{DD} - I_D R_D$

② $I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$ (ignore λ for now)

From here we can either

1. substitute ② into ① for the full large signal behavior.

* 2. Assume V_{in} has just small changes on a nominal DC bias.

↳ Taylor series

$I_D(V_{GS})$ @ $V_{GS} = V_{GS0}$ @ V_{GS0}

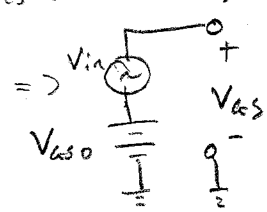
$$\frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L}(V_{in} - V_{TH}) \Rightarrow \mu_n C_{ox} \frac{W}{L}(V_{GS0} - V_{TH}) = g_m$$

$$\frac{\partial^2 I_D}{\partial V_{GS}^2} = \mu_n C_{ox} \frac{W}{L} \Rightarrow \mu_n C_{ox} \frac{W}{L} = \mathcal{B}$$

$$\Rightarrow I_D(V_{GS}) = \underbrace{\tfrac{1}{2}\mu_n C_{ox} \tfrac{W}{L}(V_{GS0} - V_{TH})^2}_{\text{DC bias}} + \underbrace{g_m(V_{GS} - V_{GS0})}_{\text{linear gain}} + \underbrace{\mathcal{B}(V_{GS} - V_{GS0})^2}_{\text{nonlinearity}}$$

(BJT will have higher order terms due to exp.)

Let $V_{in} = V_{GS} - V_{GS0}$ $\Rightarrow$ $V_{GS} = V_{in} + V_{GS0}$ $\Rightarrow$



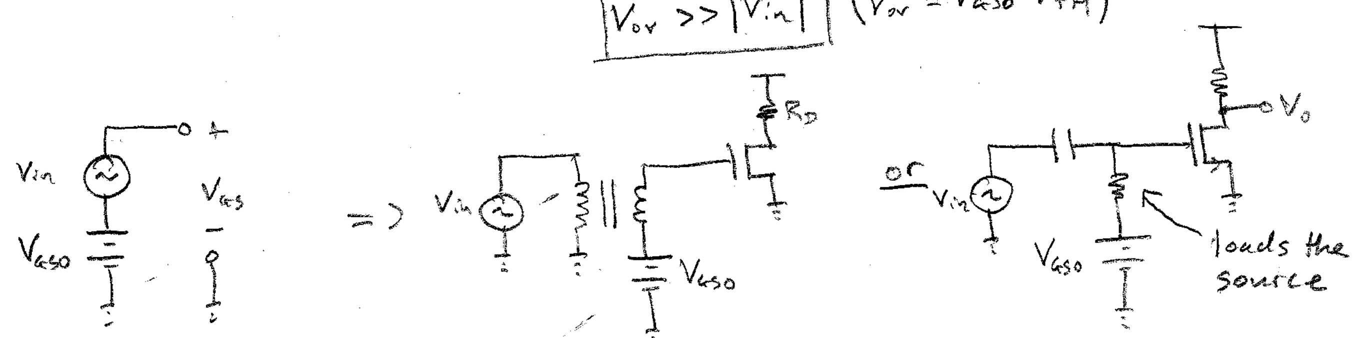
$$I_D = I_{D0} + g_m V_{in} + \mathcal{B} V_{in}^2$$

↳ ignore if

$|\mathcal{B} V_{ov} V_{in}| >> \mathcal{B} V_{in}^2$

$|V_{ov} V_{in}| >> V_{in}^2$

$\boxed{V_{ov} >> |V_{in}|}$ ($V_{ov} = V_{GS0} - V_{TH}$)

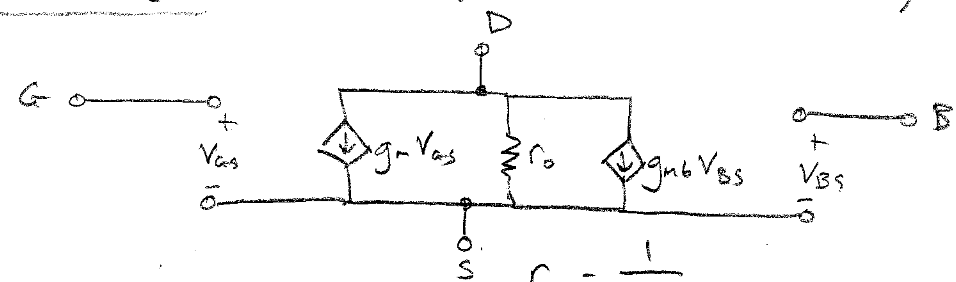


Notes:

- "small signal" and AC are not synonyms
- DC accurate amplifiers typically require differential pairs, a circuit that can take the difference of two inputs.

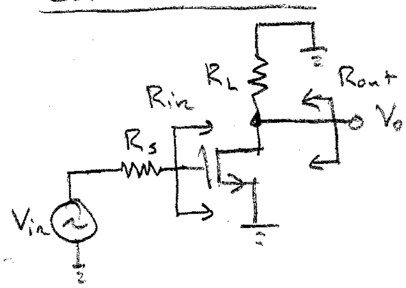
Single transistor amplifier stages (small signal, bias elements not shown)

Small signal model:



$$r_o = \frac{1}{\lambda I_D}$$

Common source



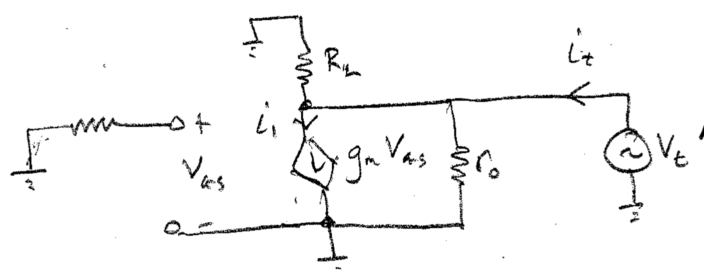
$R_{in} = \infty \Rightarrow V_{gs} = V_{in}$

$$\frac{V_o}{V_{in}} = -g_m R_{out}$$

$R_{out} = R_L \parallel r_o$

Rout calc.

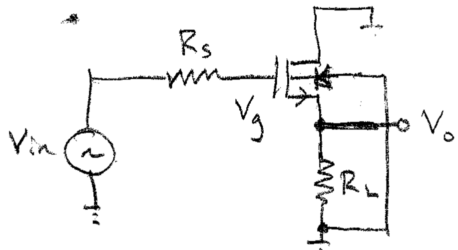
- Always set all independent sources to 0 first.



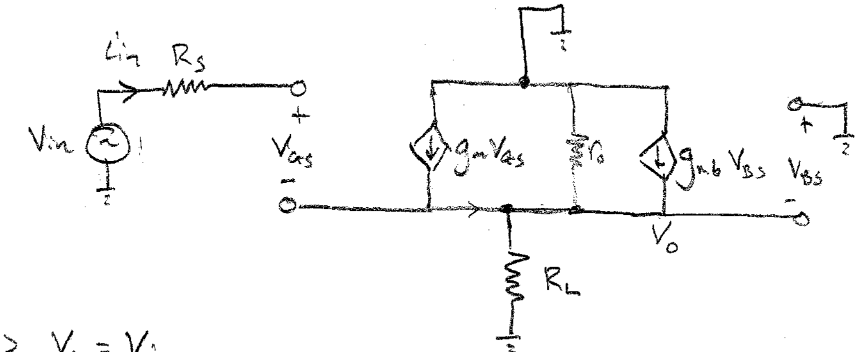
Apply a test voltage + calculate $\frac{V_t}{i_t}$

1. $V_{GS} = 0 \Rightarrow i_1 = 0$
2. $\frac{V_t}{i_t} = R_L \parallel r_o = R_{out}$

Common drain (Source follower)



$\Rightarrow$



$i_{in} = 0 \Rightarrow \boxed{R_{in} = \infty} \Rightarrow V_g = V_{in}$

KCL @ V_o

$$g_m V_{GS} - \frac{V_o}{r_o} + g_{mb} V_{BS} - \frac{V_o}{R_L} = 0$$

$V_{GS} = V_{in} - V_o$
$V_{BS} = -V_o$

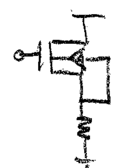
$$\Rightarrow \quad g_m (V_{in} - V_o) - \frac{V_o}{r_o} + g_{mb}(-V_o) - \frac{V_o}{R_L} = 0$$

$$V_o \left(-g_m - \frac{1}{r_o} - g_{mb} - \frac{1}{R_L}\right) = V_{in}(-g_m)$$

$$\boxed{\frac{V_o}{V_{in}} = \frac{g_m (r_o \| R_L)}{(g_m + g_{mb})(r_o \| R_L) + 1}}$$

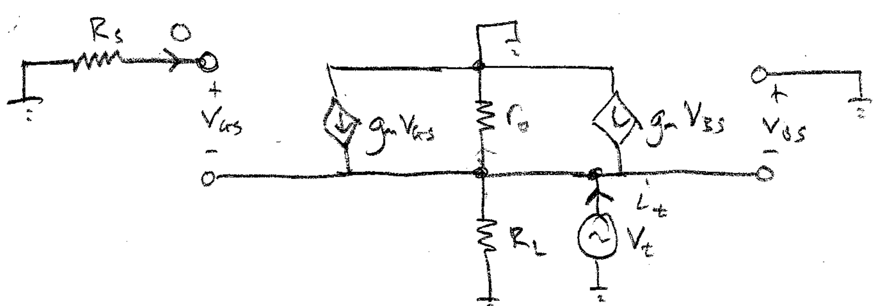
For $g_m \to \infty$ + $g_{mb} \to 0 \Rightarrow \frac{V_o}{V_{in}} \to 1$

g_{mb} degrades the gain! It would be nice if



, but this isn't possible in many cases due to processing restrictions. We would need many separate isolated substrates or wells.

R_{out}



$$i_t + g_m V_{GS} + g_{mb} V_{BS} - V_t\left(\frac{1}{r_o} + \frac{1}{R_L}\right) = 0$$

$V_{GS} = V_{BS} = -V_t$

$$i_t = V_t\left(\frac{1}{r_o} + \frac{1}{R_L} + g_m + g_{mb}\right) \Rightarrow \boxed{\frac{V_t}{i_t} = r_o \| R_L \| \frac{1}{g_m} \| \frac{1}{g_{mb}} \approx \frac{1}{g_m}}$$

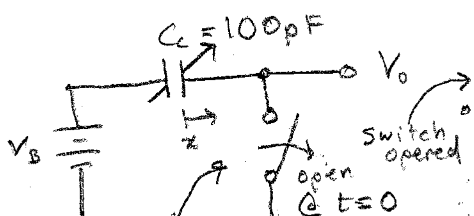
if $g_m \gg g_{mb}, \frac{1}{r_o}, \frac{1}{R_L}$

What is the point of a source follower?

- gain is 1. Is this useful?
- $R_{out} << R_{in}$.

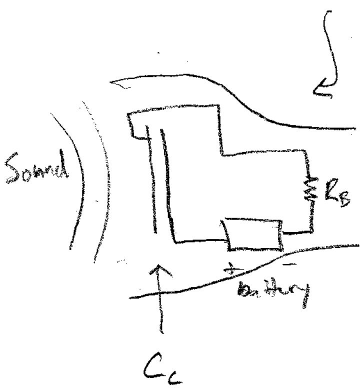
Consider an audio amplifier for driving headphones from microphone input.

condenser microphone: vibrations produce changes in capacitive gap.



- Capacitor is biased with a nominal voltage V_B creating a charge $Q = V_B C_c$
- changes in gap cause changes in cap.
- Since charge is constant, V_o changes when cap changes.

in reality, this is just a very large resistor because we don't care about DC.



$$Q = C_c(V_B - V_o)$$

$$V_o = \frac{C_c V_B - Q}{C_c} = V_B - \frac{Q}{C_c}$$

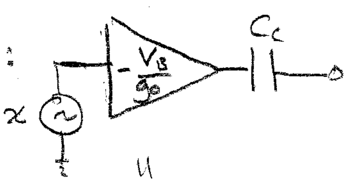
- Thevenin impedance is simply C_c

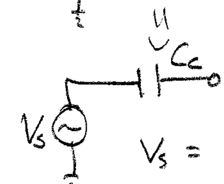
$$C_c = \frac{\varepsilon A}{g_o + x} \Rightarrow V_o = V_B - \frac{Q(x + g_o)}{\varepsilon A}$$

Small signal gain $\frac{\partial V_o}{\partial x} = -\frac{Q}{\varepsilon A} = -\frac{V_B C_c}{\varepsilon A} = -\frac{V_B}{g_o}$

$$V_o = -\frac{x}{g_o} V_B$$

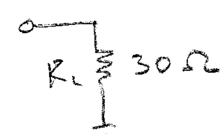
small signal model:



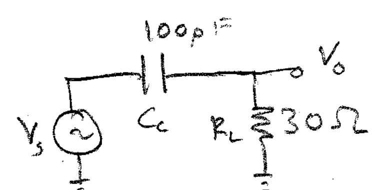


$$V_s = -\frac{x}{g_o} V_B$$

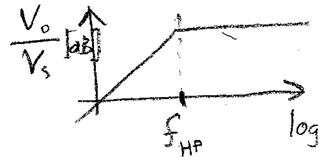
Headphones:



What happens if we connect the two?



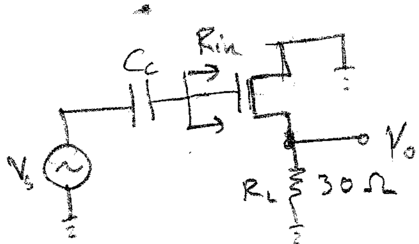
- Audio is ~20 Hz to 20 kHz
- C_c + R_L form a high pass filter



$$f_{HP} = \frac{1}{2\pi R_L C_c} = 53\ MHz!$$

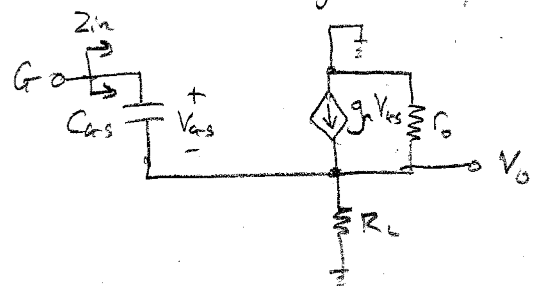
- Audio frequencies are greatly attenuated!

What if we use a follower?

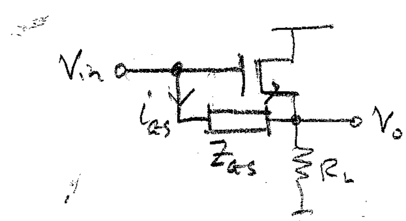


- $R_{in} = \infty$
- $V_o \approx V_s$ if $g_m R_L >> 1$
- If we have enough g_m, we can reduce the high pass filter effect by relying on large R_{in} & small R_{out}

Note: real MOSFETs have gate capacitance.



- C_{GS} means Z_{in} is finite. we'll study frequency response of amplifiers later.
- For source followers, there is another interesting advantage



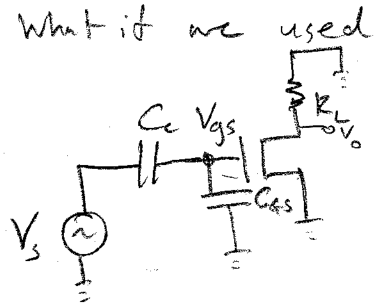
- if $g_m R_L >> 1$ $\quad V_o \approx V_{in}$

$i_{GS} = \dfrac{V_{in} - V_o}{Z_{GS}}$

As $g_m R_L \to \infty$, $i_{GS} \to 0$

1. with enough $g_m R_L$, input impedance can be arbitrarily large.
   - ignores 2nd order effects such as g_{mb}, other parasitic caps, etc.

What if we used CS?



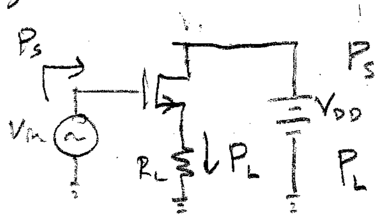
$V_{gs} = \dfrac{C_c}{C_c + C_{GS}} V_s$ => capacitance division at input.

CS is typically used in front end circuit for gain, followed by CD for load driving.

$\dfrac{V_o}{V_{in}} = -\dfrac{C_c}{C_c + C_{GS}} g_m R_L$ => if R_L is small, insert a second amplifier stage if more gain is necessary.

Observations from mic. amp design.

1. Amplifiers with voltage gain of 1 can still provide <u>power</u> gain.

Ex! 

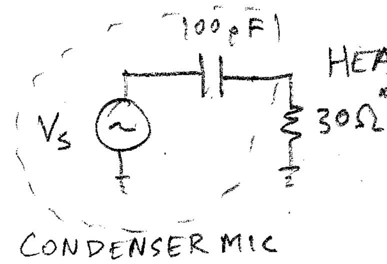
$P_s \approx 0$

$P_L = \frac{V_{in}^2}{R_L} \cdot \frac{1}{2}$

possible because $R_{out} << R_{in}$, $A_v = 1$

- Does <u>not</u> violate energy conservation. Power comes from battery. ($P_{in} = P_{out} = P_L + P_{MOSFET}$)
- Amplification = using a small signal to modulate the operating point of a higher power circuit.

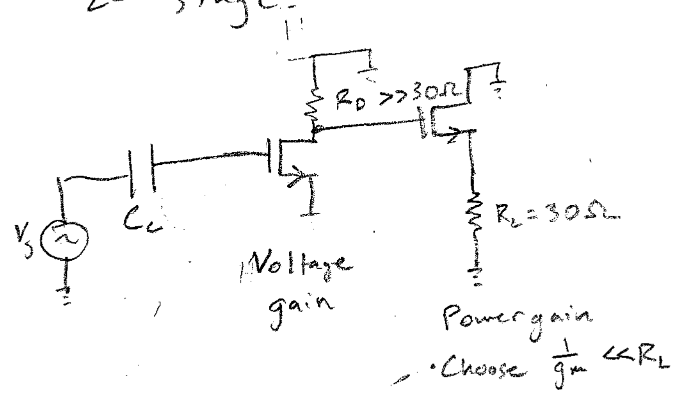
2. Interfacing to sensors with very high impedance (large R, small C) is difficult due to loading effects.



=> high pass filter effect

3. Common drain stages offer extremely high input impedance, but cannot provide voltage gain.

4. Common source stages offer high input impedance & the potential for high gain if followed by a 2nd stage.



g_m equations

g_m is defined as $\frac{\partial I_D}{\partial V_{GS}}$ (with $V_{GS}=V_{GSO}$). From $I_{Dsat} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{TH})^2$, we found $\boxed{g_m = \mu_n C_{ox} \frac{W}{L} V_{ov}}$, where

$V_{ov} = V_{GSO} - V_{TH}$ is the overdrive voltage.

For BJTs, $g_m = \frac{I_C}{V_T}$. We can come up with a similar expression for FETs.

$$\frac{I_{Dsat}}{g_m} = \frac{1}{2} V_{ov} \Rightarrow \boxed{g_m = \frac{2 I_{Dsat}}{V_{ov}}}$$

Also, $$2 I_{Dsat} \mu_n C_{ox} \frac{W}{L} = \left(\mu_n C_{ox} \frac{W}{L} V_{ov}\right)^2 = g_m^2$$

$$\therefore \boxed{g_m = \sqrt{2 I_{Dsat} \mu_n C_{ox} \frac{W}{L}}}$$

The second of these equations, $g_m = \frac{2 I_{Dsat}}{V_{ov}}$, is generally the most useful:

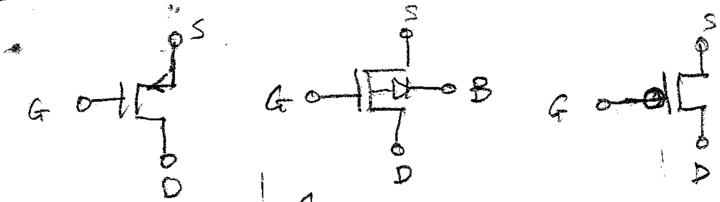
- it's independent of sizing
- V_{ov} is usually restricted by other considerations, such as linearity, noise, speed, and gain.
- It exposes the linear tradeoff between power (I_D assuming const V_{DD}) + g_m.
- It allows interesting comparisons to BJTs

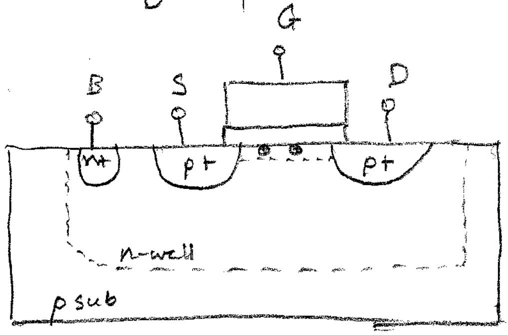
BJT	MOSFET
$g_m = \frac{I_C}{V_T}$	$g_m = \frac{2 I_D}{V_{ov}}$

Can we just make $V_{ov} < 2V_T$ in order to have a more power efficient device?

No → the square law model breaks down for $V_{ov} < \sim 100\,mV$. Known as weak inversion region: $I_D = I_S e^{V_{GS}/nV_T}$ → $n = 1.2 \ldots 1.5$

PMOS transistor





- Channel is formed by applying a negative voltage to the gate relative to S/B.
- Holes conduct current from S → D
- Bulk should be connected to V_{DD}
  - ensures parasitic diodes are off

$$I_{S \to D} = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (V_{SG} - V_{TP})^2$$

In Si $\begin{cases} \mu_p \leq 450 \frac{cm^2}{V \cdot s} \\ \mu_n \leq 1400 \frac{cm^2}{Vs} \end{cases}$ $\qquad \frac{\mu_n}{\mu_p} \approx 3 \Rightarrow$ This means that NMOS transistors are more current efficient in terms of $g_m = \sqrt{2 I_D \mu C_{ox} \frac{W}{L}}$ for identical sizing.

Note: V_{TN} + V_{TP} are usually different.

Remember:

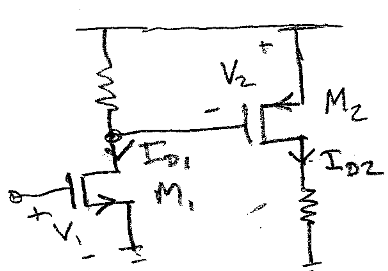
S = highest potential p+ terminal in PMOS.
It is the "source" of holes.

S = lowest potential n+ terminal in NMOS.
It is the "source" of electrons.

Current is defined as the flow of positive charge, so $I_{S \to D, PMOS}$ is <u>always</u> positive, $I_{S \to D, NMOS}$ is <u>always</u> negative.

✱ It's much less confusing to simply label currents in a circuit with a direction that intuitively makes sense.

Ex.



$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_1 - V_{TN})^2$$
$$I_{D2} = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (V_2 - V_{TP})^2$$